Implementing CCSS Mathematical Practices in the Classroom

Stephen Sher, Ed.D.
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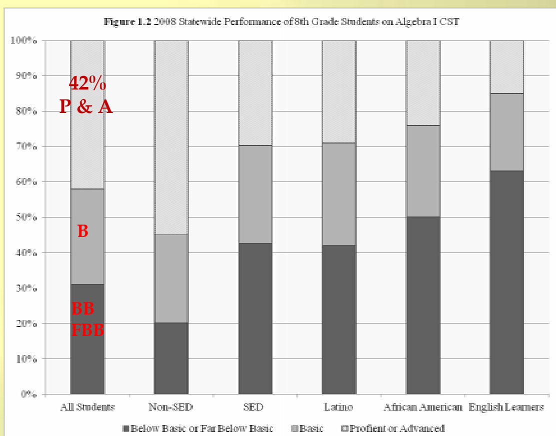
RCOE MathThink Conference

Today's Objectives

- CCSS Mathematical Practices as useful for student learning and ***making sense of mathematics***
- **Study**- Understand the CCSS Mathematical Practices through its origins or building blocks-
  - **NCTM** ***Process Standards*** and
  - ***NRC Strands of Mathematical Proficiency***
- Look at teachers' successful implementation
  - Teachers' goals for students
  - What teachers did in the classroom
  - How students responded

8th Grade CST Algebra I (2008)



CST Algebra I (2008) Grade Level – Percent Proficiency

8th Grade	42%
9th Grade	18%
10th Grade	9%
11th Grade	5%

Successful Students in Traditional Didactic AP Calculus Classes Say

Those disappointed with mathematics

We knew how to do it. But we didn't know why we were doing it… We just plugged into it- *Kate*

I think that I'm a more creative person, I can do it and understand it, but it's not something that I could do for the rest of my life and I think that if I had a job I'd like one that let me be a bit more creative- *Cathy*

I'm just not interested in just you give me a formula, I'm supposed to memorize the answer, apply it and that's it- *Kristina*

Those that liked mathematics

Because I don't really think about how or why something is. I just like math because it is or it isn't- *Jerry*

Its mathematical methodology that leads to one answer that you can get, that there's no answer other than that- *Rich*

(Boaler & Greeno, 2000)

Isolated Knowledge

Education is the acquisition of the art of the utilisation of knowledge

…in training a child to activity of thought, above all things we must beware of what I will call ***"inert ideas"***—that is to say, ideas that are merely received into the mind without being utilised, or tested, or thrown into fresh combinations.

Alfred North Whitehead *Aims of Education* (1929, p. 4)

Inert Knowledge– A Metaphor

Noble (Inert) Gases

…**rarely react with other elements**; conduct electricity, odorless, colorless, and are used in many conditions when a stable element is needed to maintain a safe and constant environment.

Chemistry Quick Facts http://www.chemistry.patent-invent.com/chemistry/noble_gases.html

CCSS for Mathematics

Practices and Content

demands students learn for understanding and explain and justify their understanding

CCSS M Grade 4 Content Standards p.30

6.Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. **Illustrate** and **explain** …**equations, rectangular arrays, and/or area models.**

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1. **Explain why** …by using **visual fraction models**…
2. Compare … by using a **visual fraction model**…
3. **Understand** a fraction a/b with a > 1 as a sum of fractions 1/b…

a.Understand addition and subtraction of fractions as …

b.Decompose a fraction …. **Justify** …**by using a visual** fraction…

d.Solve word problems … by using **visual fraction models and equations** …

4b. **Understand** a multiple of a/b as a multiple of 1/b, and use this… **Solve word problems** … by using **visual fraction models and equations to represent the problem.**

c.For example, if each person at a party will eat 3/8 of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed?

Study Purpose and Perspective

How do we get kids to understand-process/think about/reason/apply/appreciate mathematics?

Q1- **How** can sense-making mathematics be effectively *integrated* into the curriculum?

Q2- **Could it better** prepare students (for algebra)?

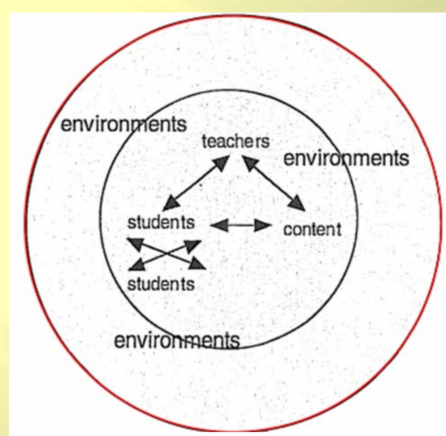
Traditional vs. Reform-Based Instruction

How knowledge is taught and learned affects the way in which it can be used

Traditional	Reform-Based
Memorization	**Understanding**
Learning as Acquisition	**Learning as Participation**
Passively receive information	Actively engage with information
Instrumental Understanding	Relational Understanding

Boaler & Greeno, (2000); Sfard (1998); Skemp (1976)

INSTRUCTION AS INTERACTION OF TEACHERS, STUDENTS, AND CONTENT IN ENVIRONMENTS



Cohen, D. K. & Ball, D. L., 2000

Learning– Theoretical Basis

A pragmatic approach, no one theoretical perspective is adequate

Constructivist- Individual Meaning

Socio-cultural- Interacting as a member of a group

Standards for Mathematical Practice (p.6)

These practices rest on important "processes and proficiencies" with longstanding importance in mathematics education. The first of these are the **NCTM process standards** .... The second are the **strands of mathematical proficiency**

CCSS– How Do We Get There?

Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

CCSS & NCTM Process Standards

NCTM Process Standards and the CCSS Mathematical Practices

NCTM Process Standards	CCSS Mathematical Practices
Problem Solving	1. Make sense of problems and persevere in solving them. 5. Use appropriate tools strategically.
Reasoning and Proof	2. Reason abstractly and quantitatively. 3. Critique the reasoning of others. 8. Look for and express regularity in repeated reasoning
Communication	3. Construct viable arguments
Connections	6. Attend to precision. 7. Look for and make use of structure
Representations	4. Model with mathematics.

J. Michael Shaughnessy, NCTM President (Nov. 4, 2011)

Correlating Standards

The Starting Point...

NCTM Processes	CCSS – Standards for Mathematical Practice	Adding it Up – Strands of Mathematical Proficiency
Problem Solving	*1. Make sense of problems and persevere in solving them.*	Strategic competence
Reasoning and Proof	*2. Reason abstractly and quantitatively.*	Adaptive reasoning
Reasoning and Proof	*3. Construct viable arguments and critique the reasoning of others.*	Adaptive reasoning
Connections	*4. Model with mathematics.*	Strategic competence
Representation	*5. Use appropriate tools strategically.*	Strategic competence Conceptual understanding
Communication	*6. Attend to precision.*	Procedural fluency.
Connections	*7. Look for and make use of structure.*	Strategic competence
Reasoning and Proof	*8. Look for and express regularity in repeated reasoning.*	Adaptive reasoning
		*Productive disposition

ems&tl Elementary Mathematics Specialists & Teacher Leaders Project — The Brookhill FOUNDATION

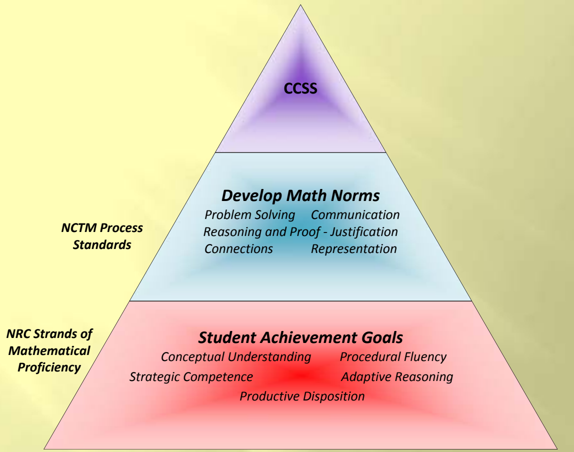
Grouping the practice standards

1. Make sense of problems and persevere in solving them
6. Attend to precision

2. Reason abstractly and quantitatively 3. Construct viable arguments and critique the reasoning of others	Reasoning and explaining
4. Model with mathematics 5. Use appropriate tools strategically	Modeling and using tools
7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.	Seeing structure and generalizing

William McCallum The University of Arizona — Standards for Mathematical Practice

Implementing CCSS Mathematical Practices



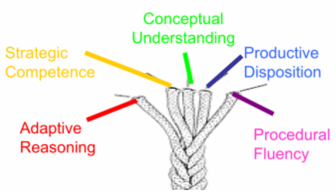
CCSS Building Blocks

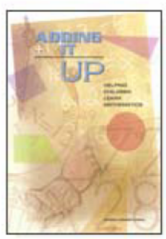
Identify Outcomes and the Means to Achieve Them

- *Strands of Mathematical Proficiency* from Adding It Up, Helping Children Learn Mathematics. (National Research Council, 2001)
- *NCTM Process Standards* from the Principles and Standards for School Mathematics (NCTM, 2000)

Underlying Frameworks

Strands of Mathematical Proficiency





What is "*necessary for anyone to learn mathematics successfully*"

NRC (2001). *Adding It Up*. Washington, D.C.: National Academies Press.

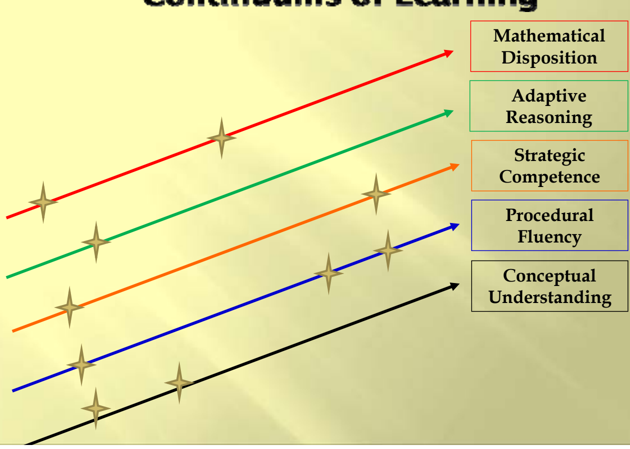
http://www.carnegielearning.com/webinars

Strands of Mathematical Proficiency

1. Conceptual Understanding of Mathematical	2. Procedural Fluency- Skill in carrying out procedures	3. Strategic Competence- Ability to	4. Adaptive Reasoning- Capacity for	5. Productive Disposition- Habitual inclination to see mathematics as
a. Concepts	a. Flexibly	a. Formulate	a. Logical thought	a. Sensible
b. Operations	b. Accurately	b. Represent	b. Reflection	b. Useful
c. Relations	c. Efficiently	c. Solve	c. Explanation	c. Worthwhile
	d. Appropriately	**Mathematical problems**	d. Justification	**Belief in**
				d. Diligence
				e. One's own efficacy

Adapted from *Adding It Up: Helping Children Learn Mathematics (NRC, 2001)*

Strands of Proficiency Continuums of Learning

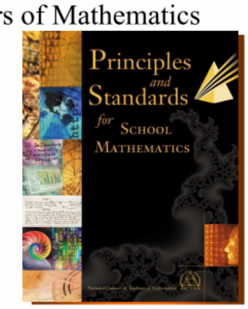


Underlying Frameworks

National Council of Teachers of Mathematics

5 **Process** Standards

- Problem Solving
- Reasoning and Proof
- Communication
- Connections
- Representations



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J. Michael Shaughnessy, NCTM President (Nov. 4, 2011)

NCTM Process Standards

Problem solving	Reasoning & Proof	Communication	Connections	Representation
Build new mathematical knowledge through problem solving	Recognize reasoning and proof as fundamental aspects of mathematics	Organize and consolidate their mathematical thinking through communication	Recognize and use connections among mathematical ideas	Create and use representations to organize, record, and communicate mathematical ideas
Solve problems that arise in mathematics and in other contexts	Make and investigate mathematical conjectures	Communicate their mathematical thinking coherently and clearly to peers, teachers, and others	Understand how mathematical ideas interconnect and build on one another to produce a coherent whole	Select, apply, and trans-late among mathematical representations to solve problems
Apply and adapt a variety of appropriate strategies to solve problems	Develop and evaluate mathematical arguments and proofs	Analyze and evaluate the mathematical thinking and strategies of others;	Recognize and apply mathematics in contexts outside of mathematics	Use representations to model and interpret physical, social, and mathematical phenomena
Monitor and reflect on the process of mathematical problem solving	Select and use various types of reasoning and methods of proof	Use the language of mathematics to express mathematical ideas precisely		

Problem Solving

What makes a *good* problem?

- More than applying a procedure
- Multiple strategies for solving
- Vehicle for other process standards
- Opportunity to learn by doing
- Application in the real world
- It is or is made relevant

Problem Solving

Build new mathematical knowledge through problem solving

Solve problems that arise in mathematics and in other contexts

Apply and adapt a variety of appropriate strategies to solve problems

Monitor and reflect on the process of mathematical problem solving

Reasoning and Proof

Convince yourself;

Convince a friend;

Convince an enemy.

(Mason, Burton, and Stacey, 1982, in Schoenfeld, Teaching and Learning Proof, Across the Grades, 2008)

Reasoning and Proof

Recognize reasoning and proof as fundamental aspects of mathematics

Make and investigate mathematical conjectures**

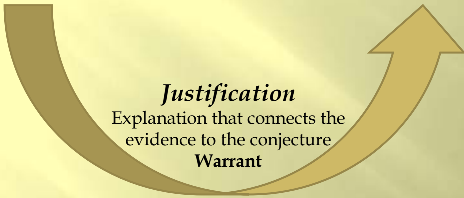
Develop and evaluate mathematical arguments and proofs

Select and use various types of reasoning and methods of proof

Reasoning and Proof – Argumentation
Claim – Grounds – Warrant

Evidence- Facts
What is Known
Grounds

Conjecture
Which can be proved or disproved
Claim



Justification
Explanation that connects the evidence to the conjecture
Warrant

Based on Stephen Toulmin's Model of Argumentation

Claim and Grounds

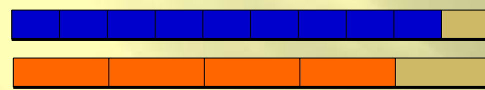
Claim - I think that $\frac{9}{10}$ is larger than $\frac{4}{5}$

Grounds -

1.) $\frac{90}{100} > \frac{80}{100}$

2.) $0.9 > 0.8$

3.)



4.)

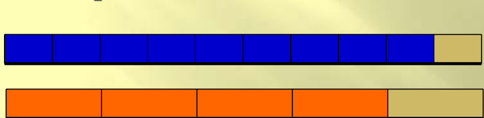
Warrant– Justification

I think that 9/10 is larger than 4/5 because

1.) $\frac{90}{100} > \frac{80}{100}$ Equivalent fractions, hundredths, reveal that 90 hundredths or $\frac{9}{10}$ is greater than 80 hundredths or $\frac{4}{5}$

2.) $0.9 > 0.8$ Equivalent decimals, tenths, reveal the same conclusion

3.)



$\frac{9}{10}$

$\frac{4}{5}$

4.)

$\frac{1}{10}$ $\frac{1}{5}$

If both quantities are looked at as *1 minus some amount* and compared, 9/10 will be closer to the quantity of 1 than 4/5 because *1 minus the smaller tenth* must be larger than *1 minus the larger fifth*.

Communication

I just need to know it, not talk about it!

Why Communicate?

- To force and expose the process of thinking
- To *transfer* the work of understanding to students
- Student thinking must be valued to be successful

Communication

Organize and consolidate their mathematical thinking through communication

Communicate their mathematical thinking coherently and clearly to peers, teachers, and others

Analyze and evaluate the mathematical thinking and strategies of others

Use the language of mathematics to express mathematical ideas precisely

Discourse-Based Instructional Tools
Chapin and O'Connor

- **Revoicing** by both teacher and students- restating and asking if the restatement is correct
- Teacher initiated request that a student repeat a previous contribution by another student
- Teacher's elicitation of student's reasoning
- Teachers request for students to add on
- Teacher wait time

Chapin, S. H. and O'Connor, C. (2007)

Connections

Recognize and use connections among mathematical ideas

Understand how mathematical ideas interconnect and build on one another to produce a coherent whole

Recognize and apply mathematics in contexts outside of mathematics

Connections

What might they look like?

- Outside mathematics- Using math to understand costs, comparing interest rates, time, resources...
- Within mathematics- How is multiplication connected to addition? Division? Base ten?
  - Between equivalent yet different mathematical representations …
  - Between fractions and decimals

Representation

Create and use representations to organize, record, and communicate mathematical ideas

Select, apply, and translate among mathematical representations to solve problems

Use representations to model and interpret physical, social, and mathematical phenomena

Representation

Why not just stick to the math?

It is math

- To understand and describe relationships between quantities- e.g. fractions- linear, area, objects as part of a group models
- Because we often better understand situations and relationships involving quantities in other ways than numbers

From Earnest, D. & Balti, A. A. (2008, May) TCM

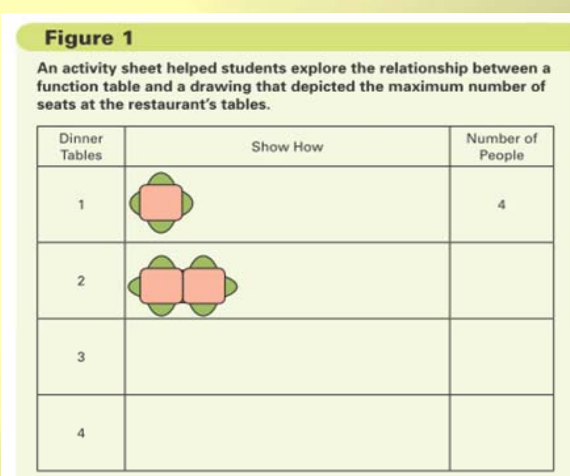
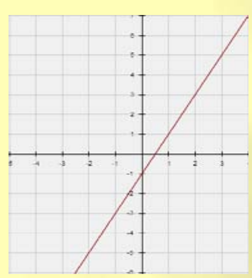


Figure 1

An activity sheet helped students explore the relationship between a function table and a drawing that depicted the maximum number of seats at the restaurant's tables.

Dinner Tables	Show How	Number of People
1		4
2		
3		
4		

Multiple Representations



$y = 2x - 1$

X	Y
-5	-11
-3	-7
-2	-5
-1	-3
0	-1
1	1
2	3
3	5
4	7
5	9

(-5, -11), (-3, -7), (-2, -5), (0, -1), (1, 1), (4, 7)

Representation

Start with Student invented representations and connect it to standard representations

The ***connections that students make*** are valuable, NOT the actual representation

Strong Implementation of Process Standard

Process Standards	Lesson	Lesson	Lesson	Lesson	Lesson	Lesson	Lesson	Lesson
Problem Solving	X		X	X	X	X	X	X
Reasoning & Proof	X	X	X	X	X	X	X	X
Communication	X	X	X	X	X	X	X	X
Connections	X	X			X		X	X
Representation	X	X	X		X	X		X

Activity

Warm-up using the distributive property

- 5 x 10 = 50
- 5 x 7 = <u>35</u>
- 5 x 17 =
- 3 x 18 =
- 10 x 17 =

Teacher Talk
Explain strategies…
Tell me more…
Does that work for all numbers?
Another example would be…
What did ____ say?
Did you do it another way?
Could you do it another way?
Do you agree with_____?

Wait Time

Activity

Warm-up- T-Chart activity

Number of Pounds	Cost
1	**$1.40**
2	
4	
8	**$11.20**
10	
20	
28	

Activity

Thanksgiving Activity

REMEMBER: Students do not know the algorithm for double digit multiplication

How long should we bake the turkey (350°)?

•The instructions say to cook the turkey at 375° for 15 minutes for every pound of weight.

•The turkey weighs 24 pounds

Adapted from *The Big Dinner* (2007) by Catherine Twomey Fosnot, Contexts for Learning Mathematics, http://firsthand.heinemann.com

Activity

You may not use algorithm for double digit multiplication

How long should we bake a 24 lb. turkey (350°) if the instructions say to cook the turkey 15 minutes for every pound of weight?

Work in pairs or triads, come to consensus on everything if possible.

-This ***is not*** a division of labor activity.

-If after group discussion you are stuck you may ask another group.

Your answers must include

•*Listing* each step in answering the question with a

•*Written explanation for* your thinking for each step.

•*Show computations* for each step.

•A *graphic representation* of thinking (chart, diagram, number line)

•All members of each group should be prepared to explain their work to the class

Adapted from *The Big Dinner* (2007) by Catherine Twomey Fosnot, Contexts for Learning Mathematics, http://firsthand.heinemann.com

Questions

- **Which Process Standard sub-areas did you use** or do you think that 4th graders would use in this activity? *Was it the same for each person?*
- **Which Strands of Mathematical Proficiency** are supported/promoted in this lesson?
- What do you need to do to integrate Process Standards into lessons?

NCTM Process Standards

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Study Findings Q1

K-5 District Policy

2 days per week or equivalent time

- CGI
- Contexts for Learning- Fosnot

4th Grade Classroom Observations, Interviews, & Surveys

- All teachers received CGI training
- 2 school sites
- Different implementations & training

General Lesson Format

Whole class: warm-up with *student discussions*

Whole class: Teacher introduces Problem - Class Discussion- usually included a small group/neighbor discussion

Small group (2-3) work – if necessary, could consult with another group

Teacher circulates, determines if there is a general issue that needs to be addressed- Supports groups

Selected **groups** explain their solution to **whole class** questions and discussion

Teacher guides explanation/conversation using *discourse-based tools*

Teachers Math Goals for Students

–Beliefs about what is good for students

- Become a problem solvers
- Have a positive attitude about mathematics
  - Enjoy- Alleviate fear of math
  - Engage in problem solving, reasoning and proof, and communication
- High expectations
  - Rigorous tasks
  - Allow for Student Struggle

Aware that they had to support and value these goals

Teacher Support for Students

Problem Solving

- Know how to start solving a problem
- Analyze a problem
- Make sense of a problem

- **Be confident in problem solving**
  - Be willing to approach
  - Not be intimidated
- **Be comfortable with rigor and struggle**

Teacher Actions

- Problem solving lessons accessible to students
- Sought multiple strategies
- Affective Support
- Orchestrated student discussions
- Prepared to teach the content for understanding- Pedagogical Content Knowledge
- Valued student thinking *(process standards)* by
  - Responding to it
  - Encouraging it
  - Rewarding it

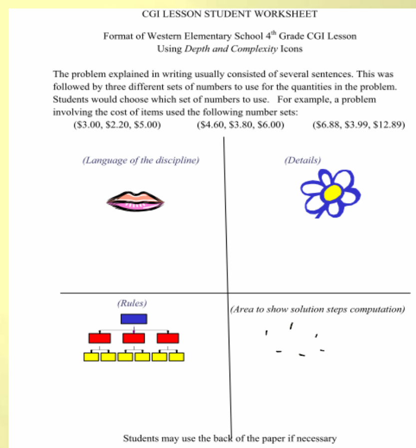
Teachers Developed Environment

- Explicitly introduced a new approach with different behaviors
- Supported behaviors through modeling, instruction, and showing they were valued
- Problem solving
- Confidence
- Integrated with familiar practice - Icons
- Teachers embraced norms and carried over some Process Standards to traditional math lessons

Students were…

- Engaged
- Struggling
- Enjoying the work
- Understanding and thinking about mathematics
  - Problem solving
  - Multiple strategies
  - Operations and procedures
  - Distributive and commutative properties
  - Students embraced and carried over some Process Standards to traditional math lessons

CGI-Icon Lesson Format



CGI LESSON STUDENT WORKSHEET

Format of Western Elementary School 4th Grade CGI Lesson
Using *Depth and Complexity* Icons

The problem explained in writing usually consisted of several sentences. This was followed by three different sets of numbers to use for the quantities in the problem. Students would choose which set of numbers to use. For example, a problem involving the cost of items used the following number sets:

($3.00, $2.20, $5.00) ($4.60, $3.80, $6.00) ($6.88, $3.99, $12.89)

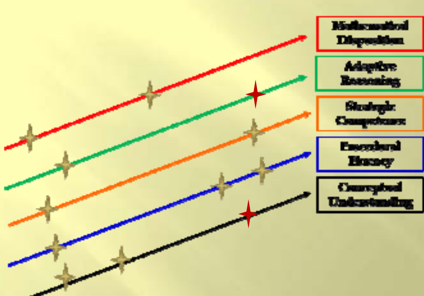
(Language of the discipline) *(Details)*

(Rules) *(Area to show solution steps computation)*

Students may use the back of the paper if necessary

Study Findings Q 2

These lessons did increase students' knowledge of mathematics and algebra readiness



Teachers are the final brokers when it comes to implementing policy

James P. Spillane, Policy Researcher

Stephen Sher

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Resources & References

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